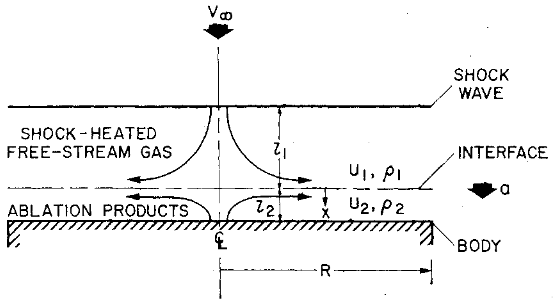

Fig. 2 Simplified physical model of forebody flowfield.

acteristic residence time at the interface is approximately 50×10^{-6} sec; the velocities u_1 and u_2 are typically 1×10^6 and 0.5×10^6 cm/sec; ρ_1 and ρ_2 are typically 10^{-6} and 10^{-5} g/cm^3; and a is approximately 10^6 cm/sec^2 (10^3 Earth g). In terms of λ, and for wavelengths of order l_2 or less, nt becomes

$$nt \sim [10^2/\lambda^2 + 10^{-2}/\lambda]^{1/2} \tag{2}$$

Clearly, from the estimate Eq. (2), Taylor instability is inconsequential when compared with shear-layer instability. Furthermore, the above computation indicates that for all λ of interest the shear-layer instability will cause disturbances initially to amplify very rapidly. This is the normal situation that eventually leads to turbulent flow in a shear layer. For short wavelengths, viscosity moderates and may even act to stabilize this growth. Quantitative results for viscous flow are, however, beyond the scope of this Note.

Since the shear-layer instability is strong, it will cause breakup and mixing of the layer and bubbles of low-density shock-heated gas will thus be trapped within the ablation products. Under the action of the buoyancy force these high-temperature bubbles will move toward the surface of the vehicle before they are swept into the wake by the bulk flow. An estimate of the distance toward the body that a bubble moves can be obtained from the solution of the following simplified equation of motion for the bubble:

$$m(dV/dt) = \underbrace{(\rho_2 - \rho_1)\tfrac{4}{3}a\pi r^3}_{\text{buoyancy force}} - \underbrace{D}_{\text{drag force}} \tag{3}$$

Here m is the mass of the bubble, r its radius, and V its velocity normal to the body. Computations show that bubble Reynolds numbers are low and that is appropriate to compute the drag for Stokes' flow; hence $D = 6\pi\mu_2(V + v_2)r$, where μ_2 is the viscosity of the ablation products (typically 10^{-3} gm/cm sec), and v_2 is the ablation-products velocity normal to the body.

A conservative estimate of the inward distance is obtained by setting $v_2 = 0$ and solving for the distance $x = V_T t$, where V_T is the terminal velocity of the bubble. This velocity is obtained by solving Eq. (3) with $dV/dt = 0$ and is given by

$$V_T = (2/9)(\rho_2 - \rho_1)ar^2/\mu_2 \tag{4}$$

With the preceding numerical values, with $t = 500 \times 10^{-6}$ sec (an overestimate of the residence time in the ablation products), and with $l_2 = 1$ cm, we obtain

$$x/r \approx r(r/l_2)^2(\text{cm})$$

Hence, the small bubbles considered here ($r < 0.1$ cm) move inward a distance less than their radius before they are swept off the body. Such motion is clearly of no concern.

In conclusion, on the basis of the above calculations for a Jovian probe, Taylor instability at the interface between shock-heated freestream gas and ablation products is inconsequential in comparison with shear-layer instability. Furthermore, the motion due to buoyancy of small bubbles of shock-heated freestream gas in the ablation products is also inconsequential. Nevertheless, the state of our knowledge concerning "massively blown" shock layers is completely unsatisfactory as regards both transition criteria and adequate models for turbulent flow. Much work, both experimental and computational, remains to be done on these flowfields.

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Evaluation of Windward Streamline Effective Cone Boundary-Layer Analyses

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Nomenclature

$C_{f\infty}$ = local skin-friction coefficient, $2\tau_w/\rho_\infty V_\infty^2$
L = slant length of sharp cone
M_∞ = freestream Mach number
$\dot{q}_w$ = wall heat flux
Re_∞/ft = unit freestream Reynolds number
$T_{o,\infty}$ = freestream stagnation temperature
T_w = wall temperature
V_∞ = freestream velocity
X_t = surface distance from sharp cone apex to onset of transition location
x = coordinate along body surface measured from apex of sharp cone
α = angle of attack
δ_v = sharp cone semivertex angle
ρ_∞ = freestream mass density
τ_w = wall shear stress

Introduction

ONE of the approximate analysis techniques in common usage among the hypersonic re-entry vehicle designers to estimate windward ray heating rates on slender cones at incidence is the so-called "effective cone" approach in which a zero angle-of-attack calculation is performed on an "effective cone" which has a cone half-angle equal to the physical cone half-angle plus the physical cone angle of attack. Needless to say, the "effective cone" technique does not properly include the effects of crossflow (outflow) as it affects the windward ray boundary-layer structure. The present Note evaluates the results of "effective cone" calculations relative to an exact three-dimensional windward ray analysis for the

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Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Boundary Layers and Convective Heat Transfer—Turbulent; Supersonic and Hypersonic Flow.

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case of laminar, transitional, and turbulent boundary-layer flow over a slender sharp cone at various angles of incidence under hypersonic cold-wall conditions.

Analysis

The present investigation makes use of the laminar, transitional, and turbulent boundary-layer analysis developed by Adams[1] for the windward streamline of a sharp cone at incidence under supersonic and hypersonic perfect gas flow conditions. The governing three-dimensional compressible turbulent boundary-layer equations in terms of time-averaged mean flow quantities as derived by Vaglio-Laurin[2] are reduced to a set of equations applicable to the windward ray (plane-of-symmetry) of a sharp cone at incidence and numerically integrated on a digital computer using a marching implicit finite-difference technique fully described in Appendix III of Ref. 1. A so-called invariant model of three-dimensional turbulence following Hunt, Bushnell, and Beckwith[3] is used in a two-layer eddy viscosity-mixing length approach for calculation of the turbulent boundary layer in conjunction with an intermittency factor treatment of the transition zone following Harris[4] and Adams.[5] The three-dimensional inviscid conical flowfield about circular cones at incidence in a perfect gas supersonic or hypersonic stream can be determined from the recent set of tables published by Jones[6] which are based upon an earlier analysis by Jones.[7] These tables are used to provide all of the needed inviscid information for input to the present boundary-layer analyses.

The zero angle-of-attack "equivalent cone" calculations reported in the present Note are based on the eddy viscosity-intermittency factor analysis by Adams[5] of transitional boundary layers on sharp cones under perfect gas hypersonic conditions. The analyses of Refs. 1 and 5 are fully compatible with respect to gas and eddy viscosity-mixing length models, as well as the intermittency treatment of transition using a factor of two ratio of transition end-to-onset distance from the sharp cone apex.

Discussion

One of the more complete experimental investigations of laminar, transitional, and turbulent boundary-layer flow over a slender sharp cone at incidence under hypersonic cold-wall wind-tunnel conditions has been reported by McCauley, Saydah, and Bueche.[8] To assess the basic accuracy of the current analysis techniques, comparisons of experimental heat-transfer rate distributions from Ref. 8 are shown in Fig. 1 relative to calculated results from the present theories. Pertinent details of the flow conditions and cone geometry are included on the figure; onset of transition locations (X_t/L) are denoted by arrows. In general, the agreement between experimental data and calculated results from the present theories is excellent for both zero and nonzero angles of incidence. The transitional heating rates are well defined by the present theories with fully turbulent flow attained near the base of the cone.

With the accuracy and applicability of the present analysis techniques now ascertained, Fig. 2 presents comparisons of "effective cone" calculations relative to the three-dimensional windward ray analysis for the case of laminar, transitional, and turbulent boundary-layer flow at various angles of attack. Note that the flow conditions are identical to those of Fig. 1 discussed previously. Also, onset to transition locations have been matched between the two analyses for a given angle of attack. As can be seen from Fig. 2, both the heat transfer and skin friction are much more sensitive to three-dimensional crossflow (outflow) effects for a laminar boundary layer than for a turbulent boundary layer. The important point here is that "effective cone" calculations appear to be acceptable for the present geometry and flow conditions, provided the boundary layer is fully turbulent. For a laminar boundary layer, "effective cone" calculations result in a 20%–30% under-prediction (based on the $x/L = 0.40$ laminar results presented in Fig. 2) for both heat transfer and skin friction

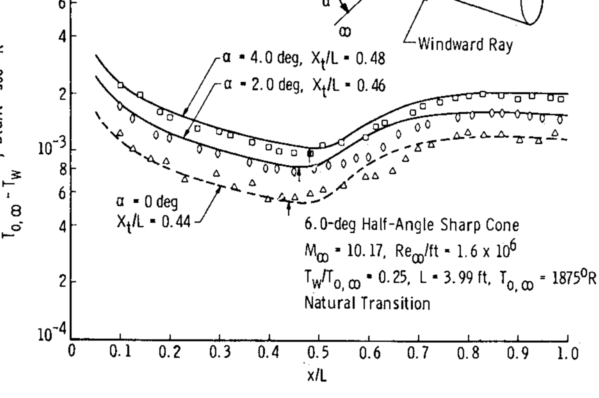

Fig. 1 Comparison of calculated and experimental windward ray heat-transfer distributions.

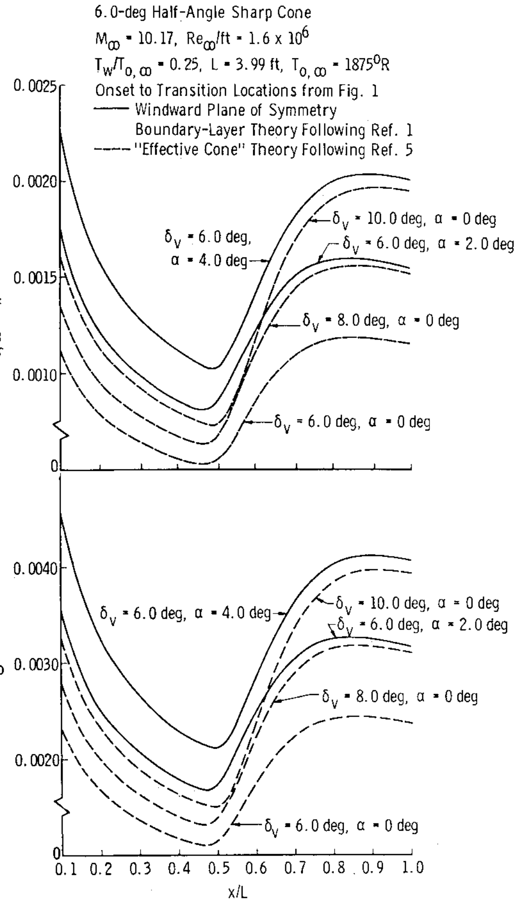

Fig. 2 Comparison of windward plane of symmetry results with "effective cone" calculations.

relative to the three-dimensional windward streamline analysis. These results confirm the statement of Vaglio-Laurin[2] that "due to the larger shearing stresses, smaller three-dimensional effects can be expected for turbulent layers as compared with laminar layers subject to the same boundary conditions."

Summary

The effects of three-dimensional crossflow (outflow) along the windward streamline of a sharp cone at incidence under hypersonic flow conditions are shown to be significantly stronger in a laminar boundary layer relative to a turbulent boundary layer. This finding is used in conjunction with an "effective cone" concept to show that windward ray heat-transfer and skin-friction distributions for a turbulent boundary layer can be computed satisfactorily (for the geometry and flow conditions under present examination) using an "effective cone" of semivertex angle equal to the physical body semivertex angle plus the physical angle of attack in a two-dimensional zero angle-of-attack analysis. Application of this "effective cone" concept to the windward streamline laminar boundary layer resulted in a rather severe under-prediction of heat transfer and skin friction relative to the windward plane-of-symmetry analysis. In general, smaller crossflow (outflow) effects on the windward streamline boundary layer can be expected for turbulent layers as compared with laminar layers subject to the same boundary conditions. It is important to realize and acknowledge that "effective cone" techniques do not yield uniformly valid approximations but may provide acceptable results in some cases (such as the present plane-of-symmetry turbulent boundary layer). Indiscriminate use of the "effective cone" approach, especially for laminar boundary layers, is not advised by the present author.

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Errata

Errata: "Thermodynamic Properties of Hydrogen-Helium Plasmas"

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[J. Spacecraft Rockets, **9**, 177–181 (1972)]

IN the definition of the quantities $\bar{C}_p$, $\bar{C}_v$, $\bar{h}$, $\bar{s}$, and $\bar{u}$ in the Nomenclature the R should be replaced by R_0, where $R_0 = R_u/M_0$. Equations (35) and (36) should be

$$C_v = C_p - R[1 - T\sum_{i=1}^{6} M_i X_{i,T|P}/\bar{M}]^2/[1 + P\sum_{i=1}^{6} M_i X_{i,P|T}/\bar{M}] \tag{35}$$

$$a/a_0 = \left[\frac{Z\gamma T}{\gamma_0 T_0}\right]^{1/2}[1 + P\sum_{i=1}^{6} M_i X_{i,P|T}/\bar{M}] \tag{36}$$

Reference 6 should be Nelson, H. F., "Thermodynamic Properties of Hydrogen-Helium Plasmas," CR-1861, 1971, NASA.

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